

Understanding Terrorist Organizations with a Dynamic Model

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Abstract

Terrorist organizations change over time because of processes such as recruitment and training as well as counter-terrorism (CT) measures, but the effects of these processes are typically studied qualitatively and in separation from each other. Seeking a more quantitative and integrated understanding, we constructed a simple dynamic model where equations describe how these processes change an organization's membership. Analysis of the model yields a number of intuitive as well as novel findings. Most importantly it becomes possible to predict whether counter-terrorism measures would be sufficient to defeat the organization. Furthermore, we can prove in general that an organization would collapse if its strength and its pool of foot soldiers decline simultaneously. In contrast, a simultaneous decline in its strength and its pool of leaders is often insufficient and short-termed. These results and other like them demonstrate the great potential of dynamic models for informing terrorism scholarship and counter-terrorism policy making.

1 Introduction

Our goal is to study terrorist organizations using a dynamic model. Generally speaking, in such a model a phenomenon is represented as a set of equations which describe it in simplified terms. The equations represent how the phenomenon changes in time or space, and cast our empirically-based knowledge in precise mathematical language. Once the model is constructed, it can be studied using powerful mathematical techniques to yield predictions, observations and insights that are difficult or impossible to collect empirically (Aris, 2005; Ellner and Guckenheimer, 2006). For example, a dynamic model could be constructed for the various militant groups operating in Iraq

and then used to predict their strength a year in the future. Moreover, given the model, it would be possible to evaluate the efficacy of various counter-insurgency policies.

Mathematical models can help fill a large methodological void in terrorism research: the lack of model systems. Whereas biologists studying pathogens can do experiments *in vitro*, there are no such model systems in terrorism research, except for mathematical models. In this sense, the method developed below provides an *in vitro* form of terrorism, which can be investigated in ways not possible in its *in vivo* kind. Like all model systems, mathematical models are imperfect because they rely on large simplifications of the underlying political phenomena, and one can rightfully ask whether their predictions would be sufficiently accurate. Fortunately, complex phenomena in fields like biology have been studied very successfully with this mathematical technique (Ellner and Guckenheimer, 2006). Therefore, even phenomena as complex as found in terrorism research may, in some cases, be productively studied using mathematical models and indeed, previous models have brought considerable insights¹.

In the rest of the paper we describe a simple model of a terrorist organization. The model is new in its focus, methodology and audience: We focus on a single terrorist organization and model its processes of recruitment, its internal dynamics as well as the impact of counter-terrorism measures on it. As to methodology, with a few exceptions (Chamberlain, 2007; Feichtinger et al., 2001; Stauffer and Sahimi, 2006; Udawadia et al., 2006) and perhaps a few others the powerful mathematical technique of differential equations has not been applied in terrorism research. Finally, the paper is specifically written to an audience of non-mathematicians: the main body of the paper uses non-technical language to explain the terminology and to describe the equations and assumptions used in the model, while the technical analysis is exposed in the appendix.

The model described below was built following two design principles. First, it was desired to have a model of broad applicability across organizations and conflicts. Indeed, the model is so general that it can be applied to insurgencies or even to some non-terrorist organizations. As we shall see, despite this generality it makes non-trivial observations and more importantly it specifies sufficient conditions for victory over the organization (see subsection 4.2). Second, it was desired to build a simple model so as to facilitate interpretation, analysis and further development. It was hoped that the

¹E.g. dynamic models: Allanach et al. (2004); Chamberlain (2007); Farley (2007); Feichtinger et al. (2001); Johnson et al. (2006); Stauffer and Sahimi (2006); Udawadia et al. (2006), rational choice models: Anderton and Carter (2005); Sandler et al. (1983); Sandler (2003); Wintrobe (2006), agent-based models: MacKerrow (2003); Tsvetovat and Carley (2007).

model would establish a methodological prototype that could be easily extended and modified to fit specific cases.

The organization of the paper is as follows. Section 2 describes the model - its variables, parameters and relations between them. Section 3 graphically illustrates the model's predictions about terrorist organizations. Sections 4 and 5 discuss the insights gleaned from the model, and the implications to counter-terrorism policies. The conclusions are in Section 6. Finally, all of the technical arguments are gathered in the appendix.

2 A Mathematical Model

There are many ways of describing a terrorist organization, such as its ideology or political platform, its operational patterns, or its methods of recruitment. Here we consider it from the “human resources” point of view. Namely, we are interested in examining how the numbers of “leaders” and “foot soldiers” in the organization change with time. The former includes experienced managers, weapon experts, financiers and even politicians and religious leaders who help the organization, while the latter are the more numerous rank-and-file. These two quantities arguably give the most important information about the strength of the organization. The precise characteristics of the two groups and their relative sizes would depend on the organization under consideration. Nevertheless, this distinction remains relevant even in the very decentralized organizations like the post-Afghanistan al-Qaeda movement, because we can identify the “leaders” as the experienced terrorists, as compared to the new recruits (see discussions in Hoffman, 2003; Sageman, 2004). The division between those two groups is also important in practice because decision makers often need to choose which of the groups to target (Wolf, 1989; Ganor, 2005, Ch.5): while the leaders represent more valuable targets, they are also harder to reach. Later on in section 5 we actually compare the policy alternatives.

Therefore, let us represent a terrorist organization as two time-varying quantities, L and F , corresponding to the number of leaders and foot soldiers, respectively. Also, L and F determine the overall “strength” S of the organization. Because leaders possess valuable skills and experience, they contribute more to the strength than an equivalent number of foot soldiers. Hence, strength S is taken to be a weighted sum of the two variables, with more weight ($m > 1$) given to leaders:

$$S = mL + F$$

We now identify a set of processes that are fundamental in changing the numbers of leaders and foot soldiers. These processes constitute the mathematical model. While some of them are self-evident, others could benefit from quantitative comparison with data. The latter task is non-trivial given the scarcity of time-series data on membership in terrorist organizations and hence we leave it out for future work.

The histories of al-Qaeda and other terrorist organizations (e.g. Laqueur, 2001; Harmon, 2000; Hoffman, 2006) suggest that the pool of terrorist leaders and experts grows primarily when foot soldiers acquire battle experience or receive training (internally, or in terrorist-supporting states, Jongman and Schmid, 2005). Consequently, the pool of leaders (L) is provisioned with new leaders at a rate proportional to the number of foot soldiers (F). We call this process “promotion” and label the parameter of proportionality p . This growth is opposed by internal personnel loss due to demotivation, fatigue, desertion as well as in-fighting and splintering (Horgan, 2005, Ch.6). This phenomenon is modeled as a loss of a fraction (d) of the pool of leaders per unit time. An additional important influence on the organization are the counter-terrorism (CT) measures targeted specifically at the leadership, including arrests, assassinations as well as efforts to disrupt communications and to force the leaders into long-term inactivity. Such measures could be modeled as the removal of a certain number (b) of people per unit time from the pool of leaders. CT is modeled as a constant rate of removal rather than as a quantity that depends on the size of the organization because the goal is to see how a fixed resource allocation towards CT would impact the organization. Presumably, the human resources and funds available to fight the given terrorist organization lead, on average, to the capture or elimination of a fixed number of operatives. In sum, we assume that on average, at every interval of time the pool of leaders is nourished through promotion, and drained because of internal losses and CT (see appendix, equation (1).)

The dynamics of the pool of foot soldiers (F) are somewhat similar to the dynamics of leaders. Like in the case of leaders, some internal losses are expected. This is modeled as the removal of a fraction (d) of the pool of operatives per unit time where for simplicity the rate d is the same as the rate for leaders (the complex case is discussed in subsection 5.2.) Much like in the case of leaders above, counter-terrorism measures are assumed to remove a fixed number (k) of foot soldiers per unit time. Finally and most importantly, we need to consider how and why new recruits join a terrorist organization. Arguably, in many organizations growth in the ranks is in proportion to the strength of the organization, for multiple reasons: Strength determines the ability to carry out successful operations, which increase interest in the organization and its mission. Moreover, strength gives the organization the manpower to publicize its attacks,

as well as to man and fund recruitment activities. By assuming that recruitment is proportional to strength, we capture the often-seen cycle where successful attacks lead to greater recruitment, which leads to greater strength and more attacks. Overall, the pool of foot soldiers is nourished through recruitment, and drained because of internal losses and CT (see appendix, equation (2))^{2,3}.

The numerical values of all of the above parameters (p, d, b, r, m, k) are dependent on the particular organization under consideration, and likely vary somewhat with time⁴. Fortunately, it is possible to draw many general conclusions from the model without knowing the parameter values, and we shall do so shortly. Finally, it should be noted that counter-terrorism need not be restricted to the parameters b, k (removal of leaders and foot soldiers, respectively), and measures such as public advocacy, attacks on terrorist bases, disruption of communication and others can weaken the organization by reducing its capabilities as expressed through the other parameters.

In the above description, we assumed that counter-terrorism measures are parameters that can be changed without affecting recruitment. This is a significant simplification because in practice it may be difficult to respond to terrorist attacks without engendering a backlash that actually promotes recruitment (see e.g. Ganor, 2008; Hanson and Schmidt, 2007). Nevertheless, the advantages of this simplification outweigh the disadvantages: Firstly, it is clear that any model that would consider such an effect would be much more complicated than the current model and consequently much harder to analyze or use. Secondly, the current model can be easily extended to incorporate such an effect if desired. Thirdly, the strength of this effect is difficult to describe in general because it depends extensively on factors such as the specific CT measures being used, the terrorist actions and the political environment. Indeed, Udawadia et al. (2006) who incorporated this effect, constructed their model based on observations of a specific context within the current conflict in Iraq.

The model includes additional implicit assumptions. First, it assumes a state of stable gradual change, such that the effect of one terrorist or counter-terrorism operation is

²A minor assumption in our model is that once a foot soldier is promoted a new foot soldier is recruited as a replacement. It is shown in the appendix that if in some organizations such recruitment is not automatic, then the current model is still valid for these organizations as long as $p < r$. In any case the drain due to promotion is marginal because foot soldiers are far more numerous than leaders even in relatively “top heavy” organizations.

³This model is similar to structured population models in biology, where the foot soldiers are the “larvae” and the leaders are the “adults”. However, an interesting difference is that whereas larvae growth is a function of the adult population alone, in a terrorist organization the pool of foot soldiers contributes to its own growth.

⁴The simplest approach to estimating them would be to estimate the number and leaders and foot soldiers at some point in time, and then find the parameter values by doing least-squares fitting of the model to the data on the terrorist attacks, where we consider the terrorist attacks to be a proxy of strength. However, this approach has some limitations.

smoothed. This should be acceptable in all cases where the terrorist organization is not very small and thus changes are not very stochastic. Second, the model assumes that an organization's growth is constrained only by the available manpower, and factors such as money or weapons do not impose an independent constraint. Third, it is assumed that the growth in foot soldiers is not constrained by the availability of potential recruits - and it is probably true in the case of al-Qaeda because willing recruits are plentiful (for the case of England, see Manningham-Buller, 2006). We discuss this point further in subsection 4.3.

3 Analysis of the Model

Having written down the governing equations, the task of studying a terrorist organization is reduced to the standard problem of studying solutions to the equations. Because the equations indicate rates of change in time, the solutions would be two functions, $L(t)$ and $F(t)$, giving the number of leaders and foot soldiers, respectively, at each time. Let us suppose that currently (time 0) the organization has a certain number of leaders and foot soldiers, L_0 and F_0 , respectively and is subject to certain CT measures, quantified by b and k . We want to see whether the CT measures are adequate to defeat the organization. Mathematically, this corresponds to the question of whether at some future time both L and F would reach zero. Intuitively, we expect that the organization would be eliminated if it is incapable of recovering from the losses inflicted on it by CT. In turn, this would depend on its current capabilities as well as the parameters p, d, r, m which characterize the organization.

Mathematical analysis of the model (see the appendix) shows that most terrorist organizations⁵ evolve in time like the organizations whose "orbits" are displayed in Fig. 1(a,b). In Fig. 1(a) we plotted eight different organizations with different starting conditions. Another perspective can be seen in Fig. 1(b) which graphically illustrates the dynamical equations via arrows: the direction of each arrow and its length indicate how an organization at the tail of the arrow would be changing and at what rate. By picking any starting location (L_0, F_0) and connecting the arrows, it is possible to visually predict the evolution into the future. Another illustration is found in Fig. 2, which shows how two example organizations change with time.

⁵That is, those with realistically low rates of desertion: $d < \frac{1}{2}(r + r\sqrt{1 + \frac{mp}{r}})$. A higher rate of desertion d always causes the organization to collapse and is not as interesting from a policy perspective (see subsection 5.2 for a discussion of desertion).

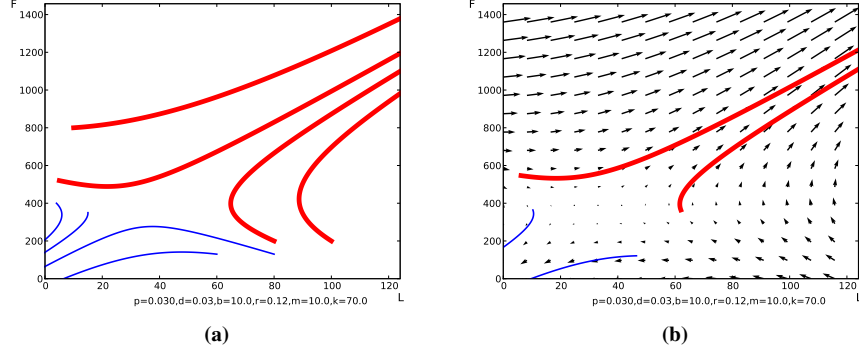


Figure 1 – (a) Typical solution curves of the equations coded by ultimate fate: thin blue for successfully neutralized organizations and thick red for those remaining operational and growing. The parameters were set to representative values, but as was said earlier, all realistic organizations are qualitatively similar and resemble these. (b) “Vector field” of L and F . At each value of L , F the direction and length of the arrow give the rate of change in L and F .

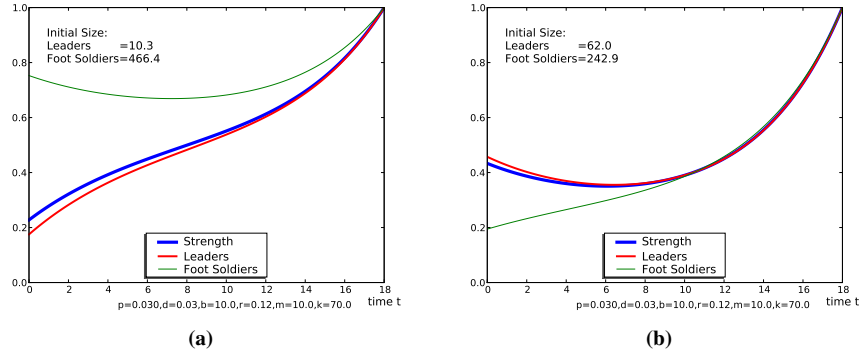


Figure 2 – Evolution of strength, leaders and foot soldiers (S , L , F , respectively) in two terrorist organizations as a function of time. In (a), due to CT, F falls initially but eventually the organization recovers through promotion. In (b), L and S fall initially but eventually the organization recovers through recruitment. The vertical axis has been rescaled by dividing each quantity by the maximum it attains during the time evolution. This makes it possible to represent all three quantities on the same plot. The units of time are unspecified since they do not affect the analysis. Of course, in a more complex model it would be desirable to consider periodic events like election cycles or generational changes.

In general, it is found that the dynamics of the organization is dependent upon the position of the organization with respect to a threshold line, which can be termed the

“sink line”: an organization will be neutralized if and only if its capabilities are below the sink line. In other words, the current CT measures are sufficient if and only if the organization lies below that threshold (thick red line on Fig. 3). The threshold is impassable: an organization above it will grow, and one below it is sure to collapse. This threshold is also very sharp: two organizations may lie close to the line, but the one above it would grow, while the one below it would shrink even if the differences in initial capabilities are small. In addition to the sink line, the model also predicts that all successful organizations would tend towards a particular trajectory. This “trend line” (a dashed black line on Fig. 3) is discussed further in subsection 4.1.

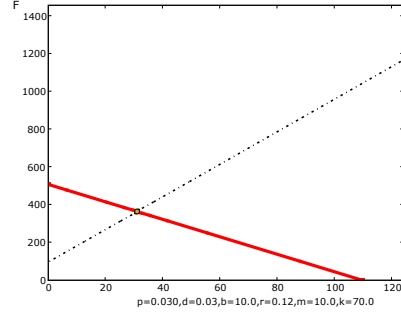


Figure 3 – Plot of the sink (thick red) and trend lines (thin dashed black). The two lines intersect at a “saddle point”.

Suppose now that the model predicts that the given organization is expected to grow further despite the current CT measures, and therefore increased CT measures would be needed to defeat it. To see the effect of additional CT measures, we need to examine how the dynamical system changes in response to increases in the values of the parameters, in particular, the parameters b and k which express the CT measures directed at leaders and foot soldiers, respectively (Fig. 4).

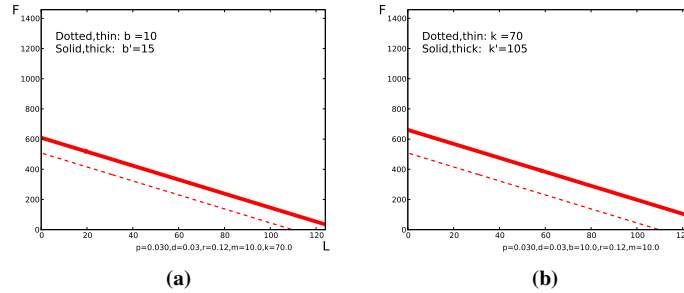


Figure 4 – The effects of the parameters b and k on the dynamical system, (a) and (b) respectively, as seen through the effect on the sink line. In each case, as the CT measures are increased, the sink line moves up confining below it additional terrorist organizations.

It is also possible to affect the fate of the organization by influencing the values of other parameters affecting its evolution, such as recruitment and promotion (Fig. 5). In general, to bring the terrorist organization under control it is necessary to change the parameters individually or simultaneously so that the organization's current state, (L, F) , is trapped under the sink line. An interesting finding in this domain is that both b and k are equivalent in the sense that both shift the sink link up in parallel (Fig. 4).

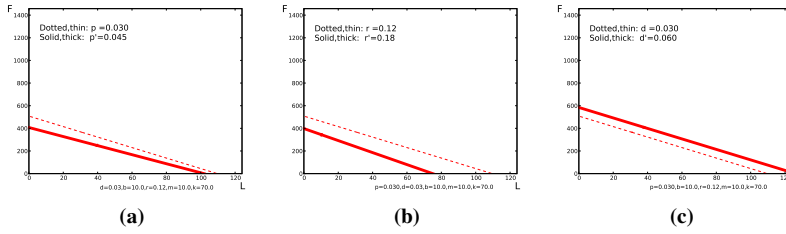


Figure 5 – The effects of the parameters p (a), r (b) and d (c) on the dynamical system as seen through the effect on the sink line. When p or r are increased the organizations are able to grow faster, causing the sink line to move down, making the existing CT measures no longer sufficient to neutralize some terrorist organizations. In contrast, when d is increased, the sink line moves up because the organization is forced to replace more internal losses to survive.

4 Discussion

4.1 Nascent terrorist organizations

Recall that the sink line (Fig. 3) distinguishes two classes of terrorist organizations - those destined to be neutralized and those that will continue growing indefinitely. Within the latter group, another distinction is introduced by the trend line - a distinction with significance to counter-terrorism efforts: organizations lying to the left of it have different initial growth patterns compared to those lying to the right (Fig. 1). The former start with a large base of foot soldiers and a relatively small core of leaders. In these organizations, F may initially decline because of CT, but the emergence of competent leaders would then start organizational growth (e.g. Fig. 2(a)). In contrast, the latter type of organizations start with a large pool of leaders but comparatively few recruits. CT could decimate their leadership, but they would develop a wide pool of foot soldiers, recover and grow (e.g. Fig. 2(b)). Thus, all successful terrorist organizations may be classified as either “p-types” (to the left of the trend line) or “r-types”

(to the right of the trend line) in reference to the parameters p of promotion and r of recruitment. In p-type organizations early growth occurs mainly through promotion of their foot soldiers to leaders, while in the r-types mainly through recruitment of new foot soldiers.

This classification could be applied to many actual organizations. For example, popular insurgencies are clearly p-type, while al-Qaeda's history since the late 1990s closely follows the profile of an r-type: Al-Qaeda may be said to have evolved through three stages: First, a core of followers moved with bin Laden to Afghanistan. They were well-trained but the organization had few followers in the wider world (for a history see Wright, Wright). Then the attacks and counter-attacks in the Fall of 2001 reduced the organization's presence in Afghanistan leaving its operatives outside the country with few leaders or skills. Finally the organization cultivated a wide international network of foot soldiers but they were ill-trained as compared to their predecessors. This description closely matches the profiles in Fig. 1 where r-type organizations start from a small well-trained core, move toward a smaller ratio of leaders to foot soldiers and then grow through recruitment.

As was noted, nascent organizations tend towards the trend line, regardless of how they started (Fig. 1). The slope of this line is $\frac{r + \sqrt{r^2 + 4rmp}}{2p}$, and this number is the long-term ratio between the number of foot soldiers and the number of leaders. Notice that this formula implies that ratio is dependent on just the parameters of growth - r, m, p - and does not depend on either d or the CT measures k, b . This ratio is generally not found in failing organizations, but is predicted to be ubiquitous in successful organizations. It may be possible to estimate it by capturing a division of an organization and it can help calculate the model's parameters. However, it is important to note that L includes not just commanding officers, but also any individuals with substantially superior skills and experience. The existence of the ratio is a prediction of the model, and if the other parameters are known, it could be compared to empirical findings.

4.2 Conditions for Victory

Recall, that the model indicates that all terrorist organizations belong to one of three classes: r-types, p-types and organizations that will be defeated. Each class exhibits characteristic changes in its leaders, foot soldiers and strength (L, F and S resp.) over time. This makes it possible to determine whether any given organization belongs to the third class, i.e., to predict whether it would be defeated.

One finding is that if a terrorist organization weakens, i.e. shows a decline in its strength S , it does not follow that it would be defeated. Indeed, in some r-type orga-

nizations it is possible to observe a temporary weakening of the organization and yet unless counter-terrorism (CT) measures are increased, the organization would recover and grow out of control (see Fig. 2(b)). Even a decline in the leadership is not by itself sufficient to guarantee victory. The underlying reason for this effect is out-of-control growth in F , which would ultimately create a new generation of terrorist leaders. Similarly, it is possible for an organization to experience a decline in its pool of foot soldiers and yet recover. These cases indicate that it is easy during a CT campaign to incorrectly perceive apparent progress in reducing the organization as a sign of imminent victory.

Fortunately, under the model it is possible to identify reliable conditions for victory over the organization (see the appendix for the proof):

1. *For a p -type organization, it is impossible to have a decline in strength S . If such a decline is made to happen, the organization would be defeated.*
2. *For an r -type organization, it is impossible to have a decline in foot soldiers F . If such a decline is made to happen, the organization would be defeated.*

Consequently:

A terrorist organization would collapse if counter-terrorism measures produce both: (1) a decline in its strength S and (2) a decline in its foot soldiers F .

In a notable contrast, declines in strength and the *leaders* are *not* sufficient in all cases (see Fig. 2(b)). To apply the theorem to an organization of an unknown type, one needs merely to estimate whether the organization's pool of foot soldiers and strength are declining. The latter could be found indirectly by looking at the quantity and quality of terrorist operations. It is not necessary to know the model's parameters or changes in the pool of leaders - the latter could even be increasing. Furthermore, while it may take some time to determine whether S and F are indeed declining, this time could be much shorter compared to the lifetime of the organization. Therefore, the theorem suggests the following two-step approach:

1. Estimate the scale of CT measures believed to be necessary to defeat the organization.
2. Measure the effect on S and F . If they both declined, then sustain the scale of operations (i.e. do not reduce b, k); Otherwise an increase in CT measures would be necessary.

The theorem and findings above give sufficient conditions for victory but they do not characterize the only possible victory scenario. For example, it is possible for an organization to see an increase in its pool of foot soldiers F yet ultimately collapse: these are organizations that lie to the right of the trend line and just slightly under the sink line. More generally, it should be remembered that to prove the theorem it was necessary to use a simplified model of a terrorist organization, as described in section 2. Nevertheless, it is likely that some form of the theorem would remain valid in complicated models because the model is built on fundamental forces that are likely to be retained in these models.

4.3 Stable Equilibria

Recall that the model does not have a stable equilibrium (Fig. 3). Yet, in many practical cases, terrorist organization seem to reach a stable equilibrium in terms of their structure and capabilities. It is plausible that such stability is the result of a dynamic balance between the growing terrorist organization and increasing CT measures directed against it. Indeed, rather than staying constant numbers like b, k , CT may actually grow when the organization presents more of a threat⁶. Aside from CT, stability may be the result of organizations reaching an external limit on their growth - a limit imposed by constraints such as funding, training facilities or availability of recruits. The case of funding could be modeled by assuming that the growth of the organization slows as the organization approaches a maximum point, (L_{max}, F_{max}) . Alternatively, it is quite possible and consistent with the model that there would be a perception of stasis because the organization is changing only slowly.

5 Counter-Terrorism Strategies

Recall that the general counter-terrorism (CT) strategy in this model is based on the location of the sink line, which we want to place above the terrorist organization (in Fig. 1). To implement this strategy, it is necessary first to calculate the model's parameters for a given organization (p, r, m, d) , and second, to determine the efficacy of the current counter-terrorism measures (b, k) . Then, it remains "just" to find the most efficient way of changing those parameters so as to move the sink line into the desired location. Let us now consider several strategic options.

⁶It would be a straightforward task to modify the model to incorporate such a control-theoretic interaction, but the task is more properly the subject of a follow-up study.

5.1 Targeting the leaders

An important “counter terrorist dilemma” (Ganor, 2005) is whom to target primarily - the leaders or the foot soldiers. Foot soldiers are an inviting target: not only do they do the vital grunt work of terrorism, they also form the pool of potential leaders, and thus their elimination does quiet but important damage to the future of the organization. Moreover, in subsection 4.1 we saw that while an organization can recover from a decline in both its strength and leadership pool, it cannot recover from declines in both its strength and its foot soldiers pool. That finding does not say that attacking leaders is unlikely to bring victory - indeed, they form an important part of the organization’s overall strength, but it does suggest that a sustained campaign against an organization is more likely to be successful when it includes an offensive against its low-level personnel. Yet, it seems that the neutralization of a terrorist leader would be more profitable since the leader is more valuable to the organization than a foot soldier, and his or her loss would inevitably result in command and control difficulties that may even disrupt terrorist attacks.

When we use the model to address the problem quantitatively, we find that the optimal strategy is actually dependent upon the organization, that is to say the parameters p, d, r, m (but not on b, k). For example, for the parameter values used in the figures above, an increase in b gives a greater rise in the sink line than an equal increase in k . Specifically, for those parameter values every two units of b are equivalent to about ten units of k . In general, when m, r, d are high but p is low then attack on the leadership is favored, while attack on the foot soldiers is best when p is high but m, r, d are low - in agreement with intuition⁷. In the first parameter range, foot soldiers are recruited so rapidly that attacking them is futile, while in the second set leaders are produced quickly so the only strategy is to fight the foot soldiers to prevent them from becoming leaders. In any case, policy prescriptions of this kind must be applied with consideration of counter-terrorism capabilities and policy costs. Thus, while on paper a particular strategy is better, the other strategy could be more feasible.

It is often argued that counter-terrorism policies have considerable side effects. For instance, there is evidence that targeted assassinations of leaders have led terrorist organizations to escalate, in what has been called the “boomerang effect” (Crenshaw, 1996, p.125). Fortunately, the model suggests that the policy maker has useful substitutes, with possibly fewer policy side effects. As Fig. 5 shows, making recruitment (r) lower

⁷Mathematically to obtain this result we first compute the derivatives of the fixed point with respect to both b and k , then project them to the orthogonal to the sink link and then use an optimization solver to find the parameter values of the model which maximize (and minimize) the ratio between the lengths of the projections.

has an effect similar to increasing k . Likewise, decreasing the rate of promotion to leadership (p) can substitute for increasing b . This agrees with intuition: for example, in the case of the foot soldiers, growth can be contained either actively through e.g. arrests or proactively by slowing the recruitment of new operatives (through e.g. attacks on recruitment facilities or advocacy).

5.2 Encouraging desertion

Fatigue and attrition of personnel have been empirically found to be an important effect in the evolution of terrorist organizations. In interviews with captured or retired terrorists, they often complained about the psychological stress of their past work, its moral contradictions, and the isolation from relatives and friends (Horgan, 2005, Ch.6). This is part of the reason why terrorist organizations cannot remain inactive (as in a cease fire) for very long without experiencing irreplaceable loss of personnel due to loss of motivation, and many organizations even resort to coercion against desertion. Therefore, encouraging operatives to leave through advocacy or amnesties may be an effective counter-terrorism strategy.

The model introduced here brings theoretical insight into this phenomenon. One prediction of the model is that even if such desertion exceeds recruitment (i.e. $d > r$) the organization would still sustain itself as long as it has a sufficiently large rate of promotion (p) or leaders of sufficiently high caliber (m). However, if d is even greater, namely, exceeds $d = \frac{1}{2}(r + r\sqrt{1 + \frac{mp}{r}})$, then the model predicts that the organization would be destroyed regardless of starting conditions, or counter-terrorism efforts (b, k).

Organizations with lower d are, of course, also effected by desertion. Earlier, in Fig. 5 we saw how increasing d raises up the sink line. To see the phenomenon in more detail, we replaced d by two (not necessarily equal) parameters d_L and d_F for the desertion of L and F , respectively. The two parameters change the slope of the sink line: increasing d_L flattens it, while increasing d_F makes it more steep (Fig. 6). Therefore, increasing d_L could be a particularly effective strategy against nascent r-type organizations, while increasing d_F could be effective against the nascent p-types.

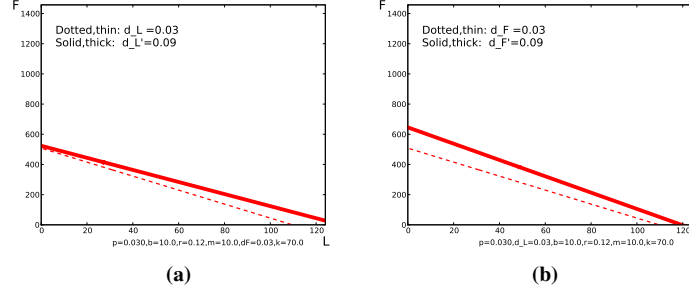


Figure 6 – The effects of d_L (a) and d_F (b) on the dynamical system, as seen through the effect on the sink line. As the desertion rates increase, the sink line moves up and its slope changes, thus trapping additional terrorist organizations.

5.3 Minimization of Strength S

Counter terrorism (CT) is often the problem of resource allocation among competing strategies. Therefore, suppose that resources have become available towards a CT operation against the terrorist organization. Namely, suppose we can remove leaders and operatives in a single blow (unlike the parameters b, k in the model which take a gradual toll). A reasonable approach to allocating these resources efficiently would be to divide them between operations targeting the leadership and those targeting the foot soldiers in such a way that the terrorist organization's strength S is minimized⁸. However, by some simplified economic analysis, it is possible to show (see appendix) that this counter-terrorism strategy is in general *suboptimal*. Instead, for a truly effective resource allocation, it is necessary to consider the dynamics of the organization being targeted and the true optimum may be considerably different. For example, when the ratio of promotion to recruitment is relatively large (i.e. $\frac{p}{r} \gg 0$), then the optimum shifts increasingly towards attacking the foot soldiers since they become much harder to replace than leaders.

On an intuitive level, the reason why the strategy is suboptimal is because often, the losses we can inflict most effectively on the organization are precisely those losses that the organization can restore most easily. Hence, in the long-term a strategy targeting strength S would be ineffective. Instead, when making a CT strategy it would be valuable to understand the target organization's dynamics, and in particular, to build a dynamical model. Such a model would help because it can identify an organization's unique set of vulnerabilities due to the composition of its human capital and its

⁸Mathematically, this would be two variable minimization of S constrained by a budget.

properties as a dynamical system.

6 Conclusions

Much of the benefit of mathematical models is due to their ability to elucidate the logical implications of empirical knowledge that was used to construct the model. Thus, whereas the empirical facts used to construct the models should be uncontroversial, their conclusions should offer new insights. The model proposed here is a very simplified description of real terrorist organizations. Despite its simplicity, it leads to many plausible predictions and policy recommendations. Indeed, the simplicity of the model is crucial to making the model useful. More detailed models of this kind could provide unparalleled insights into counter-terrorism policies and the dynamics of terrorism.

Acknowledgments

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A Appendix

The original differential equations are:

$$\frac{dL}{dt} = pF - dL - b \quad (1)$$

$$\frac{dF}{dt} = r(\underbrace{mL + F}_S) - dF - k \quad (2)$$

If we wished to incorporate the drain of the foot soldiers due to promotion ($-pF$) in Eqn.2, then we could adjust the original parameters by the transformation $r \rightarrow r - p$ and $m \rightarrow \frac{rm}{r-p}$. However, this would affect some of the analysis below, because for $r < p$ it would not longer be the case that $r > 0$, even though $rm > 0$ would still hold true. Alternatively, we could change the internal losses parameter for foot soldiers : $d_F \rightarrow d_F + p$ and break the condition $d_F = d_L$.

The linearity of the system of differential equations makes it possible to analyze the solutions in great detail by purely analytic means. The fixed point is at:

$$L_* = \frac{kp - b(r - d)}{d(r - d) + rmp} \quad F_* = \frac{kd + rmb}{d(r - d) + rmp} \quad (3)$$

The eigenvalues at the fixed point are

$$\lambda_{1,2} = \frac{r - 2d \pm \sqrt{(r - 2d)^2 + 4(rmp + d(r - d))}}{2} \quad (4)$$

From Eqn.(4), the fixed point is a saddle when $rmp + d(r - d) > 0$, i.e. $\frac{r - \sqrt{r^2 + 4rmp}}{2} < d < \frac{r + \sqrt{r^2 + 4rmp}}{2}$ (physically, the lower bound on d is 0). The saddle becomes a sink if $r < 2d$ and $rmp + d(r - d) < 0$. By Eqn.(3), this automatically gives $F_* < 0$, i.e. the organization is destroyed⁹. It is impossible to obtain either a source because it requires $r - 2d > 0$ and $rmp + d(r - d) < 0$, but the latter implies $d > r$, and so $r - 2d > 0$ is impossible; or any type of spiral because $(r - 2d)^2 + 4(rmp + d(r - d)) < 0$ is algebraically impossible¹⁰. It is also interesting to find the eigenvectors because they give the directions of the sink and trend lines:

$$e_{1,2} = \begin{pmatrix} 2p \\ r \pm \sqrt{r^2 + 4rmp} \end{pmatrix} \quad (5)$$

We see that the slope of e_2 , which is also the slope of the sink line - the stable manifold - is negative. Therefore, we conclude that the stable manifold encloses, together with the axes, the region of neutralized organizations. Concurrently, the slope of e_1 - the trend line i.e. the unstable manifold - is positive. Thus, the top half of the stable separatrix would point away from the axes, and gives the growth trend of all non-neutralized organizations ($\frac{\Delta F}{\Delta L} = \frac{r + \sqrt{r^2 + 4rmp}}{2p}$).

A.1 Proof of the theorem

Recall, we wish to show that a terrorist organization that experiences both a decline in its strength and a decline in the number of its foot soldiers will be destroyed. The proof rests on two claims: First, a p-type organization cannot experience a decline in strength, and second, an r-type organization cannot experience a decrease in F (for a

⁹Of course, the dynamical system is unrealistic once either F or L fall through zero. However, by the logic of the model, once F reaches zero, the organization is doomed because it lacks a pool of foot soldiers from which to rebuild inevitable losses in its leaders.

¹⁰The degenerate case of $\lambda = 0$ has probability zero, and is not discussed.

graphic illustration see Fig. 7 below). Thus, both a decline in strength and a decline in the number of foot soldiers cannot both occur in an r-type organization nor can they both occur in a p-type organization. Hence, such a situation can only occur in the region of defeated organizations.

As to the first claim, we begin by showing that the slope of the sink line is always greater than the slope of the iso-strength lines ($= -m$). By Eqn. (5) the slope is $\frac{r - \sqrt{r^2 + 4rmp}}{2p} = -m \frac{2}{1 + \sqrt{1 + 4\frac{mp}{r}}} > -m$. Therefore, the flow *down* the sink line has $\frac{dS}{dt} > 0$ (Down is the left-to-right flow in the figure). Now, we will show that in a p-type organization, the flow must experience an even greater increase in strength. Let A be the matrix of the dynamical system about the equilibrium point and let the state of the terrorist organization be $(L, F) = d_1 e_1 + d_2 e_2$ where e_1, e_2 are the distinct eigenvectors corresponding to the eigenvalues λ_1, λ_2 . Consideration of the directions of the vectors (Eqn.(5)) shows that for a p-type organization, $d_1 > 0$ and $d_2 < 0$. The direction of flow is therefore $d_1 \lambda_1 e_1 + d_2 \lambda_2 e_2$. Notice that $\lambda_1 > 0, \lambda_2 < 0$, and so the flow has a positive component ($= d_1 \lambda_1$) in the e_1 direction (i.e. up the trend line). Since the flow along e_1 experiences an increase in both L and F , it must experience an increase in strength. Consequently, a p-type organization must have $\frac{dS}{dt}$ which is even more positive than the flow along the sink line (where $d_1 = 0$). Thus, $\frac{dS}{dt} > 0$ for p-types.

As to the second claim, note that r-type organizations have $d_1 > 0$ and $d_2 > 0$. Moreover, in an r-type organization, the flow $d_1 \lambda_1 e_1 + d_2 \lambda_2 e_2$ has $\frac{dF}{dt}$ greater than for the flow *up* the right side of the sink line (right-to-left in the figure): the reason is that e_1 points in the direction of increasing F and while in an r-type $d_1 > 0$, along the

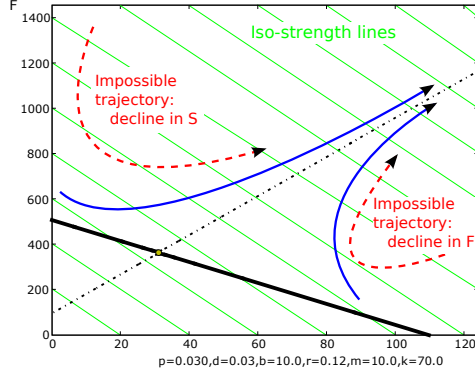


Figure 7 – The phase plane with possible (solid blue) and impossible (dashed red) trajectories, and lines of equal organization strength (green). Because orbits of the p-type must experience an increase in strength S , the left red line cannot be an orbit. Also, r-type orbits must experience an increase in F , and so the right red line cannot be an orbit either.

sink line $d_1 = 0$. The flow up the sink line has $\frac{dF}{dt} > 0$, and so $\frac{dF}{dt} > 0$ in an r-type organization. In sum, $\frac{dS}{dt} < 0$ simultaneously with $\frac{dF}{dt} < 0$ can only occur in the region $d_1 < 0$ - the region of defeated organizations. QED.

A.2 Concrete Example of Strength Minimization

In subsection 5.3 we claimed that the task of minimizing S is different from the optimal counter-terrorism strategy. Here is a concrete example that quantitatively illustrates this point. Suppose a resource budget B is to be allocated between fighting the leadership and fighting the foot soldiers, and furthermore, that the cost of removing l leaders and f foot soldiers, respectively, is a typical convex function: $c_1 l^\sigma + c_2 f^\sigma$ (c_1 and c_2 are some positive constants and $\sigma > 1$)¹¹. Notice that whereas uppercase letters L, F indicate the number of leaders and foot soldiers, respectively, we use lowercase l, f to indicate the number to be *removed*. The optimal values of l and f can be easily found graphically using the standard procedure in constrained optimization: the optimum is the point of tangency between the curve $B = c_1 l^\sigma + c_2 f^\sigma$ and the lines of constant difference in strength: $\Delta S = ml + f = \text{constant}$ (Fig.8(a)).

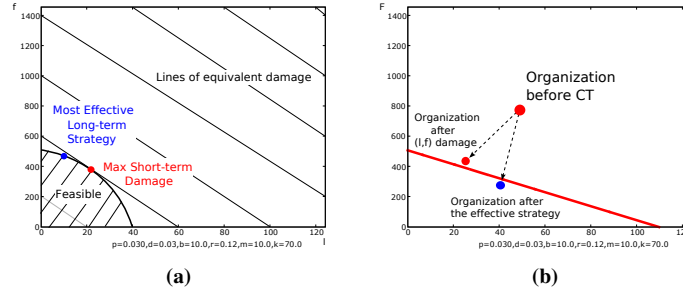


Figure 8 – Graphical calculation of optimal budget allocation (a) and contrast between minimization of S and the actual optimum (b). In (a), the optimal choice of (l, m) is given by the point of tangency between the feasible region and the lines of constant S . In (b), the red line is the sink line. The minimization of S through the removal of about 20 leaders and 400 foot soldiers would not bring the organization under the sink line, but a different (still feasible) strategy would.

However, as illustrated in Fig. 8(b), if such a strategy is followed, the terrorist organization may still remain out of control. It is preferable to choose a different strategy

¹¹ $\sigma > 1$ because e.g. once the first say 20 easy targets are neutralized, it becomes harder to find and neutralize the next 20 (the law of diminishing returns.) In any case the discussion makes clear that for most cost functions the suggested optimum would be different from the true optimum.

- in the example it is the strategy that focuses more on attacking the foot soldiers and thus brings the organization under the sink line (red line), even though the ΔS is not as large. In general, the difference between the strategies is represented by the difference between the slope of the sink line and the slopes of the lines of equivalent damage to strength. The latter always have slope $-m$ while the former becomes arbitrarily flat as $\frac{p}{r} \rightarrow \infty$.

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